College Algebra

Grade 12

Prepared by:

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Superintendent of Schools:

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Adopted by the Midland Park Board of Education on

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College Algebra

Course Description:

College Algebra is a full-year course that is designed to reinforce and expand on students' algebraic reasoning skills. College Algebra will cover topics that include linear, polynomial, exponential and logarithmic functions. In addition, this course will help to prepare students for the SAT/ACT exams, college placement tests and mathematics courses they may see in college.

Taking part in this course helps students:

1. To foster an appreciation of mathematics.
2. To observe math in the world around them.
3. To meet the New Jersey Student Learning Standards for New Jersey Public Schools.

Course Sequence:

Unit 1: Problem Solving (17 days)
Unit 2: Graphing Functions (21 days)
Unit 3: Linear and Quadratic Functions (21 days)
Unit 4: Polynomial and Rational Functions (24 days)
Unit 5: Exponential and Logarithmic Functions (22 days)
Unit 6: Sequences and Series (13 days)
Unit 7: Trigonometric Functions: (25 days)
Unit 8: Statistics and Probability: (20 days)

**The number of instructional days is an estimate based on the information available at this time. 1 day equals approximately 48 minutes of seat time. Teachers are strongly encouraged to review the entire unit of study carefully and collaboratively to determine whether adjustments to this estimate need to be made.*

Pre-requisite:

Algebra 2

Midland Park Public Schools

Unit # 1 - Overview
Content Area: College Algebra
Unit Title: Problem Solving
Grade Level: 12

Core Ideas: Students will work with different types of word problems and the ways to solve them. Students will set up equations and tables to effectively organize information in order to get to the correct solution. Working with problems presented in a variety of ways will help students with reading comprehension and finding the core of each problem efficiently.	
Unit # 1 - Standards	
Standards (Content and Technology)**:**	
CPI#:	**Statement:**
Performance Expectations (NJSLS)	
MP 1	Make sense of problems and persevere in solving them
MP 2	Reason abstractly and quantitatively
MP 3	Construct viable arguments and critique the reasoning of others
MP 4	Model with mathematics
MP 5	Use appropriate tools strategically
MP 6	Attend to precision
MP 7	Look for and make use of structure
MP 8	Look for and express regularity in repeated reasoning
Career Readiness, Life Literacies, and Key Skills	
9.2.12.CAP.5	Assess and modify a personal plan to support current interests and postsecondary plans.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition
9.4.12.TL.4	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problems
Computer Science and Design Thinking	
8.1.12.CS.2	Model interactions between application software, system software, and hardware
8.2.12.ITH.3	Analyze the impact that globalization, social media, and access to open source technologies has had on innovation and on a society's economy, politics, and culture
8.2.12.EC.2	Assess the positive and negative impacts of emerging technologies on developing countries and evaluate how individuals, non-profit organizations, and governments have responded
Intercultural Statements (Amistad, Holocaust, LGBT, etc...)	
LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ and disabled community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California,

	Berkeley and Stephen Hawking, former Director of Research at the University of Cambridge.
Amistad Law NJSA 18A:35-4.43	Explore African-American mathematicians and scientists, including but not limited to Martha Euphemia Lofton Haynes, the first African-American woman to earn a Ph.D in mathematics , and Elbert Frank Cox, the first African-American man to earn a Ph.D in mathematics in the world.
	Discuss and analyze the movie *Hidden Figures*, the story of female African-American mathematicians and engineers who worked for NASA
Holocaust Law NJSA 18A:35-28	Explore Jewish mathematicians using the article *"Jewish Mathematicians Who Changed the Course of History"* from *jewishjournal.com*
AAPI Law NJSA 18A:25-4.44	Explore Asian-American and Pacific Islander mathematicians and scientists, including but not limited to Dr. Peter Tsai, inventor of the N95 respirator and Diana Ma, data scientist and statistician for the Lakers
Companion Standards	
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words

RST.11-12.7	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
RST.11-12.8	Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
RST.11-12.9	Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
SL.11-12.4	Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose and audience.
Interdisciplinary Connection	
6.1.12.HistorySE.14.a	Explore the various ways women, racial and ethnic minorities, the LGBTQ community, and individuals with disabilities have contributed to the American economy, politics and society
6.1.12.HistorySE.14.b	Use a variety of sources from diverse perspective to analyze the social, economic and political contributions of marginalized and underrepresented groups and/or individuals.
CASEL 5 SEL Framework	

Self-Awareness	-Demonstrate honesty and integrity -Experience self-efficacy -Develop interests and a sense of purpose
Social Awareness	-Recognize strengths in others -Understand and express gratitude
Self-Management	-Identify and use stress management strategies -Exhibit self-discipline and self-motivation -Use planning and organizational skills
Relationship Skills	-Communicate effectively -Practice teamwork and collaborative problem-solving -Seek or offer support and help when needed
Responsible Decision Making	-Demonstrate curiosity and open-mindedness -Learn to make a reasoned judgment after analyzing information, data, facts -Recognize how critical thinking skills are useful both inside & outside of school

Unit Essential Question(s): ● How can we use problem solving strategies to solve different types of word problems?	**Unit Enduring Understandings:** ● Identifying the topic the question is asking about is the crucial first step in picking the correct strategy to use. ● Knowing a variety of strategies to approach problem solving is effective and efficient.

Evidence of Learning

Formative Assessments: Do Now, Homework, On-spot Checking for Understanding, Teacher Feedback **Summative/Benchmark Assessment(s):** Quizzes, Chapter Reviews, Chapter Tests
Alternative Assessments: Portfolios, Online Assignments

Resources/Materials: https://njctl.org/materials/courses/integrated-math-iii/	**Key Vocabulary:** mixture problems, rate problems, percent problems, Pythagorean Theorem, functions, data analysis

Suggested Pacing Guide

Lesson Name/Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete
Rates, Ratios, and Proportions	-Understanding how rates and ratios are related and how ratios and proportions are related -Setting up and solving proportions	Lesson, Application, Review	2 days

Mixture, Rate, and Percent Problems	-Setting up tables to organize data for mixtures, unit rates, and	Lesson, Application, Review	3 days

	percentages -Setting up and solving equations from tables		
Using the Pythagorean Theorem and Special Right Triangles	-Using the Pythagorean Theorem and knowledge of special right triangles to find the unknown -Identifying right triangles in other spatial word problems	Lesson, Application, Review	3 days
Function Word Problems	-Finding function values both graphically and algebraically -Solving word problems that incorporate functions	Lesson, Application, Review	2 days
Word Problems Involving Graphing	-Understanding when a graph is beneficial to finding the solution -Understanding the relationship of slope in real-life situations -Understanding the relationship between slope and parallel/perpendicular lines	Lesson, Application, Review	2 days
Word Problems Involving Data Analysis	-Analyzing information provided in tables and using the information to answer the given question	Lesson, Application, Review	2 days

Teacher Notes: 17 total days including assessment days (quizzes, test)

Additional Resources:

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students
-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Case Managers and follow IEP modifications/acc om modations	-Allow errors in speaking -Rephrase questions, directions, and explanations -Allow extended time on assessments	-Provide extension activities -Build on students' intrinsic motivations	-Consult with Guidance Counselors and follow I&RS procedures -Consult with classroom teacher(s) for specific behavior interventions -Provide extended time to complete tasks (on need basis)	-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Guidance Counselors and 504 Committees to come up with procedures/504 accommodations

Unit # 2 - Overview

Content Area: College Algebra	
Unit Title: Graphing Functions	
Grade Level: 12	
Core Ideas: Students will model real-life phenomena with the appropriate graphs. Students will learn which equations can be classified as functions and the different properties that relate to each family of functions.	
Unit # 2 - Standards	
Standards (Content and Technology)**:**	
CPI#:	**Statement:**
Performance Expectations (NJSLS)	
NJSLS.F-IF.1	Understand that a function from one set (called the domain) to another set (called the range assigns to each element of the domain exactly one element of the range. If ???? is a function and ???? is an element of its domain, than ????(????) denotes the output of ???? corresponding to the input ????. The graph of ???? is the graph of the equation ???? = ????(????).
NJSLS.F-IF.2	Use function notation, evaluate functions for inputs in their domain, and interpret statements that use function notation in terms of a context.
NJSLS.F-IF.7a-b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions
NJSLS.F-BF.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
Career Readiness, Life Literacies, and Key Skills	
9.2.12.CAP.5	Assess and modify a personal plan to support current interests and postsecondary plans.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition
9.4.12.TL.4	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problems
Computer Science and Design Thinking	
8.1.12.CS.2	Model interactions between application software, system software, and hardware
8.2.12.ITH.3	Analyze the impact that globalization, social media, and access to open source technologies has had on innovation and on a society's economy, politics, and culture
8.2.12.EC.2	Assess the positive and negative impacts of emerging technologies on developing countries and evaluate how individuals, non-profit organizations, and governments

	have responded
Intercultural Statements (Amistad, Holocaust, LGBT, etc...)	
LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ and disabled community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California, Berkeley and Stephen Hawking, former Director of Research at the University of Cambridge.
Amistad Law NJSA 18A:35-4.43	Explore African-American mathematicians and scientists, including but not limited to Martha Euphemia Lofton Haynes, the first African-American woman to earn a Ph.D in mathematics , and Elbert Frank Cox, the first African-American man to earn a Ph.D in mathematics in the world.
	Discuss and analyze the movie *Hidden Figures*, the story of female African-American mathematicians and engineers who worked for NASA
Holocaust Law NJSA 18A:35-28	Explore Jewish mathematicians using the article *"Jewish Mathematicians Who Changed the Course of History"* from *jewishjournal.com*
AAPI Law NJSA 18A:25-4.44	Explore Asian-American and Pacific Islander mathematicians and scientists, including but not limited to Dr. Peter Tsai, inventor of the N95 respirator and Diana Ma, data scientist and statistician for the Lakers
Companion Standards	

RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words
RST.11-12.7	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
RST.11-12.8	Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
RST.11-12.9	Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
SL.11-12.4	Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose and audience.
Interdisciplinary Connection	
6.1.12.HistorySE.14.a	Explore the various ways women, racial and ethnic minorities, the LGBTQ community, and individuals with disabilities have contributed to the American economy, politics and society
6.1.12.HistorySE.14.b	Use a variety of sources from diverse perspective to analyze the social, economic

	and political contributions of marginalized and underrepresented groups and/or individuals.
CASEL 5 SEL Framework	
Self-Awareness	-Demonstrate honesty and integrity -Experience self-efficacy -Develop interests and a sense of purpose
Social Awareness	-Recognize strengths in others -Understand and express gratitude
Self-Management	-Identify and use stress management strategies -Exhibit self-discipline and self-motivation -Use planning and organizational skills
Relationship Skills	-Communicate effectively -Practice teamwork and collaborative problem-solving -Seek or offer support and help when needed
Responsible Decision Making	-Demonstrate curiosity and open-mindedness -Learn to make a reasoned judgment after analyzing information, data, facts -Recognize how critical thinking skills are useful both inside & outside of school

Unit Essential Question(s): ● How can we express real-life phenomena using the various types of graphs? ● How do we interpret slope and the y-intercept?	**Unit Enduring Understandings:** ● Linear functions have a constant rate of change. ● Slope represent the rate of change of a function while the y-intercept represents the initial value. ● Piecewise functions are functions that be represented by different rules over different intervals.

Evidence of Learning

Formative Assessments: Do Now, Homework, On-spot Checking for Understanding, Teacher Feedback **Summative/Benchmark Assessment(s):** Quizzes, Chapter Reviews, Chapter Tests
Alternative Assessments: Portfolios, Online Assignments

Resources/Materials: https://njctl.org/materials/courses/integrated-math-iii/	**Key Vocabulary:** Linear functions, quadratic functions, absolute value functions, piecewise functions, step functions, square root functions, cube root functions

Suggested Pacing Guide

Midland Park Public Schools

Lesson Name/Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete

Vertical Line Test and Functions	-Understanding what the definition of a function is -Applying the Vertical Line Test to determine functions -Using the appropriate function notation	Lesson, Application, Review	1 day
Domain and Range	-Understand what domain and range represent -Finding domain and range of given graphs -Using interval notation to write domain and range	Lesson, Application, Review	2 days
Graphing Linear and Absolute Value Functions	-Graphing linear functions using intercepts and the slope -Interpreting the slope and y-intercept in context -Identifying the vertex of absolute value graphs -Graphing absolute value functions -Writing appropriate linear and absolute value function for a given situation	Lesson, Application, Review	2 days
Graphing Piecewise and Step Functions	-Finding the boundary points of each interval of a piecewise function -Graphing piecewise functions -Representing absolute value as a piecewise function -Graphing step functions -Writing appropriate piecewise and step functions for a given situation	Lesson, Application, Review	4 days
Graphing Quadratic Functions	-Identifying the axis of symmetry, vertex, and maxima/minima of quadratic graphs -Writing the appropriate quadratic function for a given situation	Lesson, Application, Review	3 days
Graphing Square Root and Cube Root Functions	-Graphing square root and cube root functions -Identifying turning points and end points	Lesson, Application, Review	3 days
Transformations	-Identifying the different transformations (translations, reflections, stretch/shrink) -Graphing functions using just the transformations	Lesson, Application, Review	3 days

Teacher Notes: 21 total days including assessment days (quizzes, test)

Additional Resources:

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students

-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Case Managers and follow IEP modifications/accom modations	-Allow errors in speaking -Rephrase questions, directions, and explanations -Allow extended time on assessments	-Provide extension activities -Build on students' intrinsic motivations	-Consult with Guidance Counselors and follow I&RS procedures -Consult with classroom teacher(s) for specific behavior interventions -Provide extended time to complete tasks (on need basis)	-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Guidance Counselors and 504 Committees to come up with procedures/504 accommodations

Midland Park Public Schools

Unit # 3 - Overview	
Content Area: College Algebra	
Unit Title: Linear and Quadratic Functions	
Grade Level: 12	
Core Ideas: Students will work with the families of linear and quadratic functions. The unit will cover working with different forms of the same equation, solving linear and quadratic equations with a variety of different methods, and comparing solutions of linear and quadratic equations to that of linear and quadratic inequalities. Students will learn to determine which method is appropriate based on the form of the equation given.	
Unit # 3 - Standards	
Standards (Content and Technology)**:**	
CPI#:	**Statement:**
Performance Expectations (NJSLS)	
NJSLS.N-CN.A.2	Use the relation $\square\square\square\square^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
NJSLS.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines
NJSLS.F-IF.C.7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
NJSLS.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
NJSLS.A-REI.B.4a	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the

	quadratic formula from this form.
NJSLS.A-REI.C.6	Solve systems of linear equations exactly and approximately, focusing on pair of linear equations in two variables
NJSLS.A-REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equations in two variables algebraically and graphically.
NJSLS.A-REI.B.4b	Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and imaginary numbers b
NJSLS.N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
Career Readiness, Life Literacies, and Key Skills	
9.2.12.CAP.5	Assess and modify a personal plan to support current interests and postsecondary plans.
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Responsible Decision Making	-Demonstrate curiosity and open-mindedness -Learn to make a reasoned judgment after analyzing information, data, facts -Recognize how critical thinking skills are useful both inside & outside of school

Unit Essential Question(s):	**Unit Enduring Understandings:**
● How can we use linear and quadratic functions to model real life phenomena? ● Why do we need the different but equivalent forms of these functions? ● How can we decide that the linear or quadratic function will be the best fit for a real life situation? ● Can the real number system be extended?	● Linear parent function (□□□□ = □□□□) ● Standard Form, Slope-Intercept Form, Point-Slope Form (linear functions) ● Quadratic parent function (□□□□ = □□□□^{2}) ● Real solutions of equations show the zeros of the functions which are the x-intercepts of the graphs.

● How can we solve systems of linear and quadratic functions? ● How do we use the method of completing the square to transform any quadratic expression? ● How can we solve quadratic equations by taking square roots, completing the square, the Quadratic Formula, and factoring? ● How can we solve linear and quadratic inequalities?	● Imaginary numbers (□□□□ = $\sqrt{-1}$; □□□□$^2 = -1$) ● Complex number set includes all real numbers. ● Discriminant determines number and type of solutions.

Evidence of Learning

Formative Assessments: Do Now, Homework, On-spot Checking for Understanding, Teacher Feedback **Summative/Benchmark Assessment(s):** Quizzes, Chapter Reviews, Chapter Tests
Alternative Assessments: Portfolios, Online Assignments

Resources/Materials:	**Key Vocabulary:**
https://njctl.org/materials/courses/integrated-math-iii/	System of equations, point of intersection, quadratic, factoring, square roots, completing the square, Quadratic Formula, complex number, discriminant

Suggested Pacing Guide

Lesson Name/Topic	**Student Learning Objective(s)**	**Suggested Tasks/Activities:**	**Day(s) to Complete**
Linear Functions and Their Characteristics	-Rewriting linear equations in the different forms (standard, slope-intercept, point-slope) -Identifying characteristics of linear functions	Lesson, Application, Review	2 days
Solving Systems of	-Solving systems of equations by	Lesson, Application,	3 days

Equations	substitution -Solving systems of equations by elimination	Review	
Factoring Quadratics	-Factoring quadratics completely and solving quadratics by factoring	Lesson, Application, Review	4 days
Completing the Square	-Solving quadratics by completing the square and transforming standard form into vertex form	Lesson, Application, Review	3 days
Quadratic Formula	-Solving quadratics using the Quadratic Formula -Calculating the discriminant to determine type and number of solutions	Lesson, Application, Review	2 days
Linear and Quadratic Inequalities	-Graphing linear and quadratic inequalities on a coordinate grid -Solving linear and quadratic inequalities graphically and algebraically	Lesson, Application, Review	4 days

Teacher Notes: 21 total days including assessment days (quizzes, test)

Additional Resources:

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students
-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Case Managers and follow IEP	-Allow errors in speaking -Rephrase questions, directions, and explanations -Allow extended time on assessments	-Provide extension activities -Build on students' intrinsic motivations	-Consult with Guidance Counselors and follow I&RS procedures -Consult with classroom teacher(s) for specific behavior interventions	-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Guidance Counselors and 504 Committees to come up with

modifications/accom modations			-Provide extended time to complete tasks (on need basis)	procedures/504 accommodations

Unit # 4 - Overview	
Content Area: College Algebra	
Unit Title: Polynomial and Rational Functions	
Grade Level: 12	
Core Ideas: Students will work with the family of polynomial functions. The unit will cover higher-degree polynomials and their characteristics and relating the factoring patterns of quadratics to polynomials. Students will also apply the Remainder, Factor, and Rational Theorems to polynomials to solve for zeros. Graphing calculators will be used for their CALCULATE and GRAPH features to aid the solving process. The Fundamental Theorem of Algebra will be used to further classify polynomials based on the number and type of solutions. Students will work with the family of rational functions. Rational functions are introduced using inverse and joint variation. The unit will cover the relationship between the graph of rational functions and their characteristics (domain, range, holes, asymptotes). Students will also explore how fraction operations are extended to add, subtract, multiply, and divide rational functions. Rational equations will be solved using skills from Algebra 1 (cross-products, and LCD).	
Unit # 4 - Standards	
Standards (Content and Technology)**:**	
CPI#:	**Statement:**
Performance Expectations (NJSLS)	
NJSLS.N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those value, allowing for a notation for radicals in terms of rational exponents.
NJSLS.F-IF.C.7c	Graph functions expressed symbolically and show key feature of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
NJSLS.F-IF-C.7d (+)	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior
NJSLS.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
NJSLS.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials
NJSLS.A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and and number a, the remainder on division by $x-a$ is $p(a)=0$ if and only if $(x-a)$ is a factor of $p(x)$.
NJSLS.A-APR.D.7 (+)	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
NJSLS.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

NJSLS.A-SSE.A.2	Use the structure of an expression to identify different ways to rewrite it.
NJSLS.N-CN.C.9(+)	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
Career Readiness, Life Literacies, and Key Skills	
9.2.12.CAP.5	Assess and modify a personal plan to support current interests and postsecondary plans.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition
9.4.12.TL.4	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problems
Computer Science and Design Thinking	
8.1.12.CS.2	Model interactions between application software, system software, and hardware
8.2.12.ITH.3	Analyze the impact that globalization, social media, and access to open source technologies has had on innovation and on a society's economy, politics, and culture

8.2.12.EC.2	Assess the positive and negative impacts of emerging technologies on developing countries and evaluate how individuals, non-profit organizations, and governments have responded
Intercultural Statements (Amistad, Holocaust, LGBT, etc...)	
LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ and disabled community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California, Berkeley and Stephen Hawking, former Director of Research at the University of Cambridge.
Amistad Law NJSA 18A:35-4.43	Explore African-American mathematicians and scientists, including but not limited to Martha Euphemia Lofton Haynes, the first African-American woman to earn a Ph.D in mathematics , and Elbert Frank Cox, the first African-American man to earn a Ph.D in mathematics in the world.
	Discuss and analyze the movie *Hidden Figures*, the story of female African-American mathematicians and engineers who worked for NASA
Holocaust Law NJSA 18A:35-28	Explore Jewish mathematicians using the article *"Jewish Mathematicians Who Changed the Course of History"* from *jewishjournal.com*
AAPI Law NJSA 18A:25-4.44	Explore Asian-American and Pacific Islander mathematicians and scientists, including but not limited to Dr. Peter Tsai, inventor of the N95 respirator and Diana Ma, data scientist and statistician for the Lakers
Companion Standards	

RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words
RST.11-12.7	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
RST.11-12.8	Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
RST.11-12.9	Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
SL.11-12.4	Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose and audience.
Interdisciplinary Connection	
6.1.12.HistorySE.14.a	Explore the various ways women, racial and ethnic minorities, the LGBTQ community, and individuals with disabilities have contributed to the American economy, politics and society
6.1.12.HistorySE.14.b	Use a variety of sources from diverse perspective to analyze the social, economic and political contributions of marginalized and underrepresented groups and/or individuals.
CASEL 5 SEL Framework	
Self-Awareness	-Demonstrate honesty and integrity -Experience self-efficacy -Develop interests and a sense of purpose
Social Awareness	-Recognize strengths in others -Understand and express gratitude
Self-Management	-Identify and use stress management strategies -Exhibit self-discipline and self-motivation -Use planning and organizational skills
Relationship Skills	-Communicate effectively -Practice teamwork and collaborative problem-solving -Seek or offer support and help when needed
Responsible Decision Making	-Demonstrate curiosity and open-mindedness -Learn to make a reasoned judgment after analyzing information, data, facts -Recognize how critical thinking skills are useful both inside & outside of school

Unit Essential Question(s):

- How can polynomial functions be used to model real life problems?
- How can properties of linear and quadratic functions be generalized to polynomial functions?
- What are some common characteristics of polynomial graphs?
- How do we use the factors of a polynomial to solve a division problem?
- How do we factor a polynomial?
- What is the Fundamental Theorem of Algebra?
- How do the characteristics of quadratics apply to polynomials?
- How can rational functions be used to model real-life problems?
- How are inverse variation and rational functions related?
- What do vertical/horizontal asymptotes of rational functions signify?
- How do we determine excluded values in a rational function?
- How can a rational function be solved?
- How are rational functions graphed?
- How are the four basic operations applied to rational functions?
- How do we compare the different characteristics of rational functions?

Unit Enduring Understandings:

- Definition of a polynomial function
- End behavior of polynomial graphs
- Polynomial division (Long and Synthetic)
- Algebraic properties of polynomial functions
- Degree of a polynomial tells how many roots it has (including repeated and imaginary)
- Direct variation vs. Inverse variation
- Fraction operations (addition, subtraction, multiplication, division)
- Factoring polynomials
- Local and global behaviors of rational functions
- Translations of functions (□□□□ = □□□□$_{-h}$ + □□□□)
- Domain, range, holes, asymptotes

Evidence of Learning

Formative Assessments: Do Now, Homework, On-spot Checking for Understanding, Teacher Feedback **Summative/Benchmark Assessment(s):** Quizzes, Chapter Reviews, Chapter Tests
Alternative Assessments: Portfolios, Online Assignments

Resources/Materials:
https://njctl.org/materials/courses/integrated-math-iii/

Key Vocabulary:
polynomial functions, degree, leading coefficient, end behavior, polynomial division, Remainder Theorem, Factor Theorem, Rational Zero Theorem, Fundamental Theorem of Algebra, rational functions, direct variation, inverse variation, joint variation, slant asymptotes, vertical asymptotes, holes, excluded values

Suggested Pacing Guide

Lesson Name/Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete

Polynomial Functions	-Identifying the characteristics of polynomials (including their degree, leading coefficient, and constant) -Exploring similarities and differences between even and odd degree polynomials -Graphing polynomials	Lesson, Application, Review	2 days
Add, Subtract, Multiply Polynomials	-Performing addition, subtraction, and multiplication on sets of polynomials to simplify	Lesson, Application, Review	1 days

Factoring Polynomials	-Factoring polynomials completely and solving polynomials by factoring	Lesson, Application, Review	2 days
The Rational Zero Theorem	-Using the Rational Zero Theorem to create a list of possible rational zeros -Applying the Factor Theorem to solve polynomials for their rational zeros	Lesson, Application, Review	2 days
The Fundamental Theorem of Algebra	-Applying the Fundamental Theorem of Algebra to obtain all possible zeros for a polynomial -Using Descartes's Rule of Signs to determine the number of positive and negative zeros for each polynomial	Lesson, Application, Review	3 days
Inverse and Joint Variation	-Solve direct, inverse, joint variation problems -Use the comparison of two or more units to determine inverse and joint variation	Lesson, Application, Review	1 days
Rational Function Graphs	-Graph simple rational functions -Graph translations of rational functions -Identify domain, range, holes, and asymptotes	Lesson, Application, Review	3 days
Multiply and Divide Rational Functions	-Apply fraction operations to simplify rational expression multiplication and division	Lesson, Application, Review	2 days
Add and Subtract Rational Functions	-Apply fraction operations to simplify rational expression addition and subtraction	Lesson, Application, Review	2 days
Rational Equations	-Solve rational equations using cross-products and LCD -Check for extraneous solutions	Lesson, Application, Review	3 days
Teacher Notes: 24 total days including assessment days (quizzes, test)			
Additional Resources:			

Differentiation/Modification Strategies				
Students with Disabilities	**English Language Learners**	**Gifted and Talented Students**	**Students at Risk**	**504 Students**
-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Case Managers and follow IEP modifications/acc om modations	-Allow errors in speaking -Rephrase questions, directions, and explanations -Allow extended time on assessments	-Provide extension activities -Build on students' intrinsic motivations	-Consult with Guidance Counselors and follow I&RS procedures -Consult with classroom teacher(s) for specific behavior interventions -Provide extended time to complete tasks (on need basis)	-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Guidance Counselors and 504 Committees to come up with procedures/504 accommodations

Unit # 5 - Overview	
Content Area: College Algebra	
Unit Title: Exponential and Logarithmic Functions	
Grade Level: 12	
Core Ideas: Students will work with the family of exponential and logarithmic functions. The unit will cover the inverse relationship between exponential and logarithmic functions. The properties of exponents are extended into the properties logarithms which will be used to condense and expand logarithmic expressions. Sets of data can be represented as either exponential or power functions. Students will determine whether an exponential or power function is more appropriate before writing the functions.	
Unit # 5 - Standards	
Standards (Content and Technology)**:**	
CPI#:	**Statement:**
Performance Expectations (NJSLS)	
NJSLS.F-IF.C.7e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude
NJSLS.F-BF.B.5 (+)	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents

NJSLS.F-LE.A.4	Understand the inverse relationship between exponents and logarithms.
NJSLS.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, two input-output pairs (include reading these from a table.)
Career Readiness, Life Literacies, and Key Skills	
9.2.12.CAP.5	Assess and modify a personal plan to support current interests and postsecondary plans.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition
9.4.12.TL.4	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problems
Computer Science and Design Thinking	
8.1.12.CS.2	Model interactions between application software, system software, and hardware
8.2.12.ITH.3	Analyze the impact that globalization, social media, and access to open source technologies has had on innovation and on a society's economy, politics, and culture
8.2.12.EC.2	Assess the positive and negative impacts of emerging technologies on developing countries and evaluate how individuals, non-profit organizations, and governments have responded
Intercultural Statements (Amistad, Holocaust, LGBT, etc...)	
LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ and disabled community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California, Berkeley and Stephen Hawking, former Director of Research at the University of Cambridge.
Amistad Law NJSA 18A:35-4.43	Explore African-American mathematicians and scientists, including but not limited to Martha Euphemia Lofton Haynes, the first African-American woman to earn a Ph.D in mathematics , and Elbert Frank Cox, the first African-American man to earn a Ph.D in mathematics in the world.
	Discuss and analyze the movie *Hidden Figures*, the story of female African-American mathematicians and engineers who worked for NASA
Holocaust Law NJSA 18A:35-28	Explore Jewish mathematicians using the article *"Jewish Mathematicians Who Changed the Course of History"* from *jewishjournal.com*
AAPI Law NJSA 18A:25-4.44	Explore Asian-American and Pacific Islander mathematicians and scientists, including but not limited to Dr. Peter Tsai, inventor of the N95 respirator and Diana Ma, data scientist and statistician for the Lakers
Companion Standards	

RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words
RST.11-12.7	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
RST.11-12.8	Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
RST.11-12.9	Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
SL.11-12.4	Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose and audience.
Interdisciplinary Connection	
6.1.12.HistorySE.14.a	Explore the various ways women, racial and ethnic minorities, the LGBTQ community, and individuals with disabilities have contributed to the American economy, politics and society
6.1.12.HistorySE.14.b	Use a variety of sources from diverse perspective to analyze the social, economic and political contributions of marginalized and underrepresented groups and/or individuals.
CASEL 5 SEL Framework	
Self-Awareness	-Demonstrate honesty and integrity -Experience self-efficacy -Develop interests and a sense of purpose
Social Awareness	-Recognize strengths in others -Understand and express gratitude
Self-Management	-Identify and use stress management strategies -Exhibit self-discipline and self-motivation -Use planning and organizational skills
Relationship Skills	-Communicate effectively -Practice teamwork and collaborative problem-solving -Seek or offer support and help when needed
Responsible Decision Making	-Demonstrate curiosity and open-mindedness -Learn to make a reasoned judgment after analyzing information, data, facts -Recognize how critical thinking skills are useful both inside & outside of school

Unit Essential Question(s):	**Unit Enduring Understandings:**
● How can exponential functions be used to model real life problems? ● What are some characteristics of exponential and logarithmic functions? ● What is the relationship between exponential and logarithmic functions? ● What is the natural base? ● How can the properties of exponents be used to derive the properties of logarithms? ● How can we solve exponential and logarithmic equations? ● How do we determine whether a set of data fits an exponential pattern or a power pattern?	● Exponential form <-> Logarithmic form ● Exponential and logarithmic graph translations ● Natural base ����and natural logarithm �������� ● Logarithm evaluations ● Logarithm graphs ● Exponential growth and decay models ● Compound interest and continuously compounded interest models Abstract and quantitative reasoning

Evidence of Learning

Formative Assessments: Do Now, Homework, On-spot Checking for Understanding, Teacher Feedback **Summative/Benchmark Assessment(s):** Quizzes, Chapter Reviews, Chapter Tests
Alternative Assessments: Portfolios, Online Assignments

Resources/Materials:	**Key Vocabulary:**
https://njctl.org/materials/courses/integrated-math-iii/	exponential growth/decay, compound interest, horizonal asymptote, logarithm, natural base

Suggested Pacing Guide

Lesson Name/Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete
Exponential Growth and Decay	-Graphing exponential growth and decay functions -Applying the growth and decay models to real life problems -Applying the compound interest formula to real-life problems	Lesson, Application, Review	3 days
Natural Base e	-Simplifying natural base expressions -Evaluating natural base expressions -Identifying growth and decay with natural base exponential functions -Graphing natural base exponential functions	Lesson, Application, Review	2 days
Logarithms	-Converting between exponential and logarithmic forms -Evaluating logarithms with and without a calculator -Finding inverses of logarithmic functions -Graphing logarithmic functions	Lesson, Application, Review	3 days

Properties of Logarithms	-Condensing expressions using the properties of logarithms -Expanding expressions using the properties of logarithms	Lesson, Application, Review	3 days
Exponential and Logarithmic Equations	-Solving exponential equations -Solving logarithmic equations -Checking for extraneous solutions	Lesson, Application, Review	3 days
Exponential and Power Functions	-Checking whether sets of data fit an exponential or power function -Writing exponential and power functions given a set of points	Lesson, Application, Review	3 days

Teacher Notes: 22 total days including assessment days (quizzes, test)

Additional Resources:

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students
-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Case Managers and follow IEP modifications/acc om modations	-Allow errors in speaking -Rephrase questions, directions, and explanations -Allow extended time on assessments	-Provide extension activities -Build on students' intrinsic motivations	-Consult with Guidance Counselors and follow I&RS procedures -Consult with classroom teacher(s) for specific behavior interventions -Provide extended time to complete tasks (on need basis)	-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Guidance Counselors and 504 Committees to come up with procedures/504 accommodations

Unit # 6 - Overview

Content Area: College Algebra

Unit Title: Sequences and Series

Grade Level: 12

Core Ideas: Students will study and analyze the patterns of sequences and series. They will represent sequences and series in different ways, find the nth term of different sequences, and will use summation notation. Students will compare and contrast arithmetic and geometric sequences, and will use proven formulas to find the nth term of both types of sequences.

Unit # 6 - Standards	
Standards (Content and Technology)**:**	
CPI#:	**Statement:**
Performance Expectations (NJSLS)	
NJSLS.A-SEE.B4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
NJSLS.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
Career Readiness, Life Literacies, and Key Skills	
9.2.12.CAP.5	Assess and modify a personal plan to support current interests and postsecondary plans.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition
9.4.12.TL.4	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problems
Computer Science and Design Thinking	
8.1.12.CS.2	Model interactions between application software, system software, and hardware
8.2.12.ITH.3	Analyze the impact that globalization, social media, and access to open source technologies has had on innovation and on a society's economy, politics, and culture
8.2.12.EC.2	Assess the positive and negative impacts of emerging technologies on developing countries and evaluate how individuals, non-profit organizations, and governments have responded
Intercultural Statements (Amistad, Holocaust, LGBT, etc...)	
LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ and disabled community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California, Berkeley and Stephen Hawking, former Director of Research at the University of Cambridge.
Amistad Law NJSA 18A:35-4.43	Explore African-American mathematicians and scientists, including but not limited to Martha Euphemia Lofton Haynes, the first African-American woman to earn a Ph.D in mathematics , and Elbert Frank Cox, the first African-American man to earn a Ph.D in mathematics in the world.
	Discuss and analyze the movie *Hidden Figures*, the story of female African-American mathematicians and engineers who worked for NASA
Holocaust Law NJSA 18A:35-28	Explore Jewish mathematicians using the article *"Jewish Mathematicians Who Changed the Course of History"* from *jewishjournal.com*

AAPI Law NJSA 18A:25-4.44	Explore Asian-American and Pacific Islander mathematicians and scientists, including but not limited to Dr. Peter Tsai, inventor of the N95 respirator and Diana Ma, data scientist and statistician for the Lakers
Companion Standards	
RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words
RST.11-12.7	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.

RST.11-12.8	Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
RST.11-12.9	Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
SL.11-12.4	Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose and audience.
Interdisciplinary Connection	
6.1.12.HistorySE.14.a	Explore the various ways women, racial and ethnic minorities, the LGBTQ community, and individuals with disabilities have contributed to the American economy, politics and society
6.1.12.HistorySE.14.b	Use a variety of sources from diverse perspective to analyze the social, economic and political contributions of marginalized and underrepresented groups and/or individuals.
CASEL 5 SEL Framework	
Self-Awareness	-Demonstrate honesty and integrity -Experience self-efficacy -Develop interests and a sense of purpose
Social Awareness	-Recognize strengths in others -Understand and express gratitude
Self-Management	-Identify and use stress management strategies -Exhibit self-discipline and self-motivation -Use planning and organizational skills
Relationship Skills	-Communicate effectively -Practice teamwork and collaborative problem-solving -Seek or offer support and help when needed

Responsible Decision Making	-Demonstrate curiosity and open-mindedness -Learn to make a reasoned judgment after analyzing information, data, facts -Recognize how critical thinking skills are useful both inside & outside of school

Unit Essential Question(s): ● How do you represent a sequence of numbers or the sum of a sequence? ● How to you find the nth term or partial sum of an arithmetic sequence? ● How do you find terms and sums of geometric sequences?	**Unit Enduring Understandings:** ● Represent sequences and series using the appropriate notation ● Model and find sums of arithmetic and geometric series

Evidence of Learning

Formative Assessments: Do Now, Homework, On-spot Checking for Understanding, Teacher Feedback **Summative/Benchmark Assessment(s):** Quizzes, Chapter Reviews, Chapter Tests
Alternative Assessments: Portfolios, Online Assignments

Resources/Materials: https://njctl.org/materials/courses/integrated-math-iii/	**Key Vocabulary:** Arithmetic sequence and series, geometric sequence and series, common difference, common ratio, partial sum, finite series, infinite series

Suggested Pacing Guide

Lesson Name/Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete
Sequences and Series	-Recognizing patterns in a sequence of numbers -Using the appropriate notation to represent sequences and series -Understanding the difference between sequences and series	Lesson, Application, Review	2 days

Arithmetic Sequences and Partial Sums	-Modeling arithmetic sequences -Finding the sum of arithmetic series -Applying the formula for arithmetic sequences to find unknown terms -Finding partial sums of infinite sequences	Lesson, Application, Review	4 days
Geometric Sequences and Series	-Modeling geometric sequences -Finding the sum of finite and infinite geometric series -Applying the formula for geometric sequences to find unknown terms	Lesson, Application, Review	4 days

Teacher Notes: 13 total days including assessment days (quizzes, test)

Additional Resources:

Differentiation/Modification Strategies				
Students with Disabilities	**English Language Learners**	**Gifted and Talented Students**	**Students at Risk**	**504 Students**
-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Case Managers and follow IEP modifications/acc om modations	-Allow errors in speaking -Rephrase questions, directions, and explanations -Allow extended time on assessments	-Provide extension activities -Build on students' intrinsic motivations	-Consult with Guidance Counselors and follow I&RS procedures -Consult with classroom teacher(s) for specific behavior interventions -Provide extended time to complete tasks (on need basis)	-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Guidance Counselors and 504 Committees to come up with procedures/504 accommodations

Unit # 7 - Overview
Content Area: College Algebra
Unit Title: Trigonometric Functions
Grade Level: 12
Core Ideas: Students will work with right triangle trigonometry to find the ratios of the side lengths. Students will extend right triangle trigonometry to create the unit circle and use the trigonometric functions to find and evaluate angle measures. Students will be evaluating and graphing the six trigonometric functions (sine, cosine, tangent, cosine, secant, and cotangent). The trigonometric functions will also be applied to word problems involving angles of elevation and depression.

Unit # 7 - Standards	
Standards (Content and Technology)**:**	
CPI#:	**Statement:**
Performance Expectations (NJSLS)	
NJSLS.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
NJSLS.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
NJSLS.F-TF.A.3(+)	Use special triangles to determine geometrically the values of sine, cosine, tangent for ◆◆◆◆

	$\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$ and use the unit circle to express the values of sine, cosine, and tangent for π, $\pi + \pi$, and $2\pi - \pi$ in terms of their values for π, where π is any real number.
NJSLS.F-TF.A.4(+)	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
NJSLS.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
NJSLS.F-TF.B.6(+)	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
NJSLS.F-TF.B.7(+)	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
Career Readiness, Life Literacies, and Key Skills	
9.2.12.CAP.5	Assess and modify a personal plan to support current interests and postsecondary plans.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition
9.4.12.TL.4	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problems
Computer Science and Design Thinking	
8.1.12.CS.2	Model interactions between application software, system software, and hardware
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8.2.12.EC.2	Assess the positive and negative impacts of emerging technologies on developing countries and evaluate how individuals, non-profit organizations, and governments have responded
Intercultural Statements (Amistad, Holocaust, LGBT, etc...)	
LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ and disabled community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California, Berkeley and Stephen Hawking, former Director of Research at the University of Cambridge.
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	Discuss and analyze the movie *Hidden Figures*, the story of female African-American mathematicians and engineers who worked for NASA

Holocaust Law NJSA 18A:35-28	Explore Jewish mathematicians using the article *"Jewish Mathematicians Who Changed the Course of History"* from *jewishjournal.com*
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Companion Standards	
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RST.11-12.8	Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
RST.11-12.9	Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
SL.11-12.4	Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose and audience.
Interdisciplinary Connection	
6.1.12.HistorySE.14.a	Explore the various ways women, racial and ethnic minorities, the LGBTQ community, and individuals with disabilities have contributed to the American economy, politics and society
6.1.12.HistorySE.14.b	Use a variety of sources from diverse perspective to analyze the social, economic and political contributions of marginalized and underrepresented groups and/or individuals.
CASEL 5 SEL Framework	
Self-Awareness	-Demonstrate honesty and integrity -Experience self-efficacy -Develop interests and a sense of purpose
Social Awareness	-Recognize strengths in others -Understand and express gratitude
Self-Management	-Identify and use stress management strategies -Exhibit self-discipline and self-motivation -Use planning and organizational skills
Relationship Skills	-Communicate effectively -Practice teamwork and collaborative problem-solving

	-Seek or offer support and help when needed
Responsible Decision Making	-Demonstrate curiosity and open-mindedness -Learn to make a reasoned judgment after analyzing information, data, facts -Recognize how critical thinking skills are useful both inside & outside of school

Unit Essential Question(s): ● How do you describe angles and angular movement? ● How do you evaluate trigonometric functions by using the unit circle? ● How do you use trigonometry to find unknown side lengths and angle measures in right triangles? ● How do you evaluate trigonometric functions of any angle? ● How do you sketch the graphs of sine, cosine, and other trigonometric functions?	**Unit Enduring Understandings:** ● Reciprocal and inverse are different processes. ● Trigonometric identities are true in both radian and degree modes. ● The reference angle is the acute angle formed with the horizontal axis. ● Use reference angles when evaluating trigonometric functions of angles greater than 90 degrees. ● The inverse sine function can be stated as the phrase "the angle (or number) whose sine is x".

● How do you evaluate and graph the inverses of trigonometric functions? ● How do you use trigonometric functions to solve real life problems?	● The values of the inverse sine function are always in radians. ● The range of each inverse trigonometric function is limited to allow it to be a function, and each range is different.

Evidence of Learning

Formative Assessments: Do Now, Homework, On-spot Checking for Understanding, Teacher Feedback **Summative/Benchmark Assessment(s):** Quizzes, Chapter Reviews, Chapter Tests
Alternative Assessments: Portfolios, Online Assignments

Resources/Materials: https://njctl.org/materials/courses/integrated-math-iii/	**Key Vocabulary:** Sine, cosine, tangent, radian, unit circle, reference angle, angle of elevation, angle of depression

Suggested Pacing Guide

Lesson Name/Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete
Radian and Degree Measure	-Distinguish between radian and degree measures -Convert angles to both modes	Lesson, Application, Review	1 days
Trigonometric Functions: The Unit Circle	-Use radian measures and the definitions of the trigonometric functions on the unit circle	Lesson, Application, Review	3 days
Right Triangle Trigonometry	-Find ratios of an acute angle by drawing a triangle and of any angle by drawing the unit circle and using the reference angle	Lesson, Application, Review	3 days

Trigonometric Functions of Any Angle	-Evaluate trigonometric functions of any angle, focusing on angles that are not the foundation angles on the unit circle	Lesson, Application, Review	3 days
Graphs of Sine and Cosine Functions	-Graph the sine and cosine functions -Identifythe basic characteristics of the trigonometric functions	Lesson, Application, Review	3 days
Graph of Other Trigonometric Functions	-Graphing the other trigonometric functions -Identifying the basic characteristics of the trigonometric functions	Lesson, Application, Review	3 days
Inverse Trigonometric Functions	-Evaluating inverse trigonometric functions -Graphing the inverse trigonometric functions	Lesson, Application, Review	3 days
Application and Models	-Using the trigonometric ratios to solve problems in a variety of contexts including but not limited to angles of elevation and angles of depression	Lesson, Application, Review	3 days

Teacher Notes: 25 total days including assessment days (quizzes, test)

Additional Resources:

Differentiation/Modification Strategies

Students with Disabilities	English Language Learners	Gifted and Talented Students	Students at Risk	504 Students
-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Case Managers and follow IEP	-Allow errors in speaking -Rephrase questions, directions, and explanations -Allow extended time on assessments	-Provide extension activities -Build on students' intrinsic motivations	-Consult with Guidance Counselors and follow I&RS procedures -Consult with classroom teacher(s) for specific behavior interventions	-Rephrase questions, directions, and explanations -Allow extended time on assessments -Consult with Guidance Counselors and 504 Committees

modifications/acc om modations			-Provide extended time to complete tasks (on need basis)	to come up with procedures/504 accommodations

Unit # 8 - Overview

Content Area: College Algebra

Unit Title: Statistics and Probability

Grade Level: 12

Core Ideas: Students will work on probability and odds of simple events. The unit will cover the differences between mutually exclusive and inclusive events, and independent and dependent events. Students will calculate the probabilities of events, using the Addition Rule or the Multiplication Rule depending on the type of event. Throughout the unit, students will work on finding permutations and combinations, a large part in calculating probability where both are a measure of finding groups of ����objects out of ����. With permutations, the order in which objects are picked determine a different outcome. With combinations, the order in which objects are picked do not matter. Students will explore the world of statistics through the eyes of a researcher. With each data set, students will determine type of distribution and calculate the measures of central tendency and variation accordingly.

Unit # 8 - Standards

Standards (Content and Technology)**:**

CPI#:	**Statement:**
Performance Expectations (NJSLS)	
NJSLS.S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements, of other events (“or”, “and”, “not”)
NJSLS.S-CP.B.9 (+)	Use permutation and combinations to compute probabilities of compound events and solve problems
NJSLS.S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring is the product of their probabilities, and use this characterization to determine if they are independent
NJSLS.S-CP.A.3	Understand that the condition probability of A given B as P(A and B) | P(B), and interpret independence of A and B saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B
NJSLS.S-CP.B.7	Apple the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
NJSLS.S-CP.B.8 (+)	Appy the general Multiplication Rule in a uniform probability model, P(A and B) = P(A) * P(B|A) = P(B) * P(A|B)
NJSLS.A-APR.C.5 (+)	Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal’s Triangle.
NJSLS.S-MD.A.3 (+)	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.

NJSLS.S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculator, spreadsheets, and tables to estimate areas under the normal curve.
Career Readiness, Life Literacies, and Key Skills	
9.2.12.CAP.5	Assess and modify a personal plan to support current interests and postsecondary plans.
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas
9.4.12.CI.3	Investigate new challenges and opportunities for personal growth, advancement, and transition
9.4.12.TL.4	Collaborate in online learning communities or social networks or virtual worlds to analyze and propose a resolution to a real-world problems
Computer Science and Design Thinking	
8.1.12.CS.2	Model interactions between application software, system software, and hardware
8.2.12.ITH.3	Analyze the impact that globalization, social media, and access to open source technologies has had on innovation and on a society's economy, politics, and culture
8.2.12.EC.2	Assess the positive and negative impacts of emerging technologies on developing countries and evaluate how individuals, non-profit organizations, and governments have responded

Intercultural Statements (Amistad, Holocaust, LGBT, etc...)	
LGBTQ and Disabilities NJSA 18A:35-4.35	Explore mathematicians in the LGBTQ and disabled community, including but not limited to Juliette Bruce, NSF Postdoctoral Fellow at University of California, Berkeley and Stephen Hawking, former Director of Research at the University of Cambridge.
Amistad Law NJSA 18A:35-4.43	Explore African-American mathematicians and scientists, including but not limited to Martha Euphemia Lofton Haynes, the first African-American woman to earn a Ph.D in mathematics , and Elbert Frank Cox, the first African-American man to earn a Ph.D in mathematics in the world.
	Discuss and analyze the movie *Hidden Figures*, the story of female African-American mathematicians and engineers who worked for NASA
Holocaust Law NJSA 18A:35-28	Explore Jewish mathematicians using the article *"Jewish Mathematicians Who Changed the Course of History"* from *jewishjournal.com*
AAPI Law NJSA 18A:25-4.44	Explore Asian-American and Pacific Islander mathematicians and scientists, including but not limited to Dr. Peter Tsai, inventor of the N95 respirator and Diana Ma, data scientist and statistician for the Lakers
Companion Standards	

RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words
RST.11-12.7	Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
RST.11-12.8	Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
RST.11-12.9	Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
SL.11-12.4	Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose and audience.
Interdisciplinary Connection	
6.1.12.HistorySE.14.a	Explore the various ways women, racial and ethnic minorities, the LGBTQ community, and individuals with disabilities have contributed to the American economy, politics and society
6.1.12.HistorySE.14.b	Use a variety of sources from diverse perspective to analyze the social, economic and political contributions of marginalized and underrepresented groups and/or individuals.
CASEL 5 SEL Framework	
Self-Awareness	-Demonstrate honesty and integrity -Experience self-efficacy -Develop interests and a sense of purpose
Social Awareness	-Recognize strengths in others -Understand and express gratitude
Self-Management	-Identify and use stress management strategies -Exhibit self-discipline and self-motivation -Use planning and organizational skills
Relationship Skills	-Communicate effectively -Practice teamwork and collaborative problem-solving -Seek or offer support and help when needed
Responsible Decision Making	-Demonstrate curiosity and open-mindedness -Learn to make a reasoned judgment after analyzing information, data, facts -Recognize how critical thinking skills are useful both inside & outside of school

Unit Essential Question(s): ● How can we list the possible outcomes in the sample space of an experiment?	**Unit Enduring Understandings:** ● Counting Principle of Multiplication ● Permutations

● How can we determine whether two events are independent or dependent? ● How can we find probabilities of disjoint and overlapping events? ● How can a tree diagram help us visualize the number of ways in which two or more events can occur? ● How can we determine the frequency of each outcome of an event? ● In a normal distribution, what percent of data lies within ▯▯▯▯ standard deviations of the mean? ● How can we test theoretical probability? ● What is a binomial distribution? ● How is a binomial distribution related to Pascal's Triangle?	● Combinations ● Mutually exclusive events vs. Inclusive events ● Independent events vs. Dependent events ● Complementary events ● Recognize data sets that are normal ● ▯▯▯▯-scores ● Pascal's Triangle ● Measures of Central Tendency and Variation

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Resources/Materials: https://njctl.org/materials/courses/integrated-math-iii/	**Key Vocabulary:** Probability, permutations, combinations, mutually exclusive, independent, complementary events, theoretical probability, experimental probability, normal distribution, binomial distribution, standard deviation, measures of central tendency, measures of variation, Pascal's Triangle, Binomial Theorem

Suggested Pacing Guide

Lesson Name/Topic	Student Learning Objective(s)	Suggested Tasks/Activities:	Day(s) to Complete
Probabilities and Odds	-Analyze the difference between probabilities and odds -Calculate simple probability	Lesson, Applications, Review	1 days
Probabilities using Permutations	-Apply the permutation formula (used when order matters) -Find the probabilities of events involving permutations	Lesson, Applications, Review	2 days
Probabilities using Combinations	-Apply the combination formula (used when order does not matter) -Find probabilities of events involving similar triangles	Lesson, Applications, Review	2 days

Disjoint and Overlapping Events	-Determine the difference between disjoint ("or") and overlapping ("and") <mutually exclusive vs. inclusive> -Calculate probabilities of disjoint and overlapping events	Lesson, Applications, Review	2 days
Independent and Dependent Events	-Determine the difference between independent and dependent events -Calculate probabilities of independent and dependent events	Lesson, Applications, Review	2 days
Binomial Theorem	-Use combinations to determine probabilities -Use Pascal's Triangle to find the number of combinations -Apply the Binomial Theorem to binomial expansions	Lesson, Application, Review	2 days

Measures of Central Tendency and Variation	-Identify and calculate the measures of central tendency (mean, median) -Identify and calculate the measures of variation (variance, standard deviation, range)	Lesson, Application, Review	2 days
Binomial Distributions	-Construct a probability distribution -Determine whether a probability distribution can be classified as a binomial distribution -Calculate probability of a specific number of success in a binomial distribution	Lesson, Application, Review	2 days
Normal Distributions	-Use the Empirical Rule to determine normality -Calculate area under a normal curve -Calculate the z-score for a standard normal distribution and use it to find probabilities	Lesson, Application, Review	2 days

Teacher Notes: 20 total days including assessment days (quizzes, test)

Additional Resources:

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